The internal workings of a Ducted Fan

The rotor velocity vectors and blade angles

After looking at EDFs from a pure axial change of momentum position we must now address the question how the fan is shaped inside and how the power is transferred from the motor to the air.

An EDF is normally supplied in the form shown below:

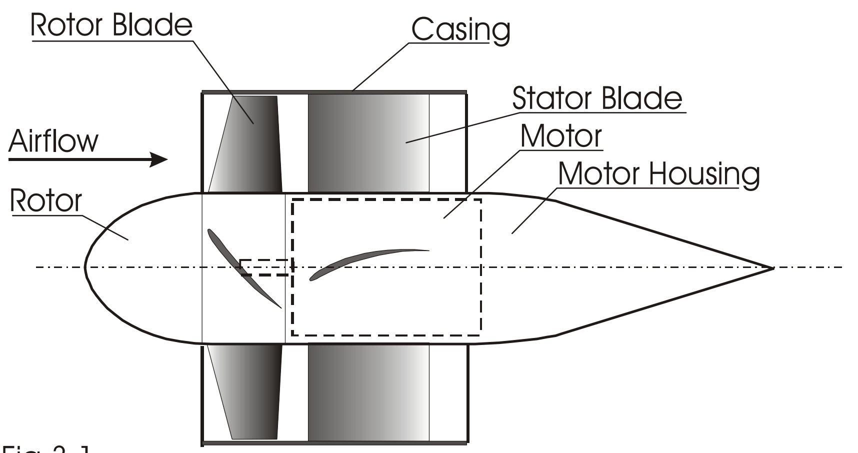


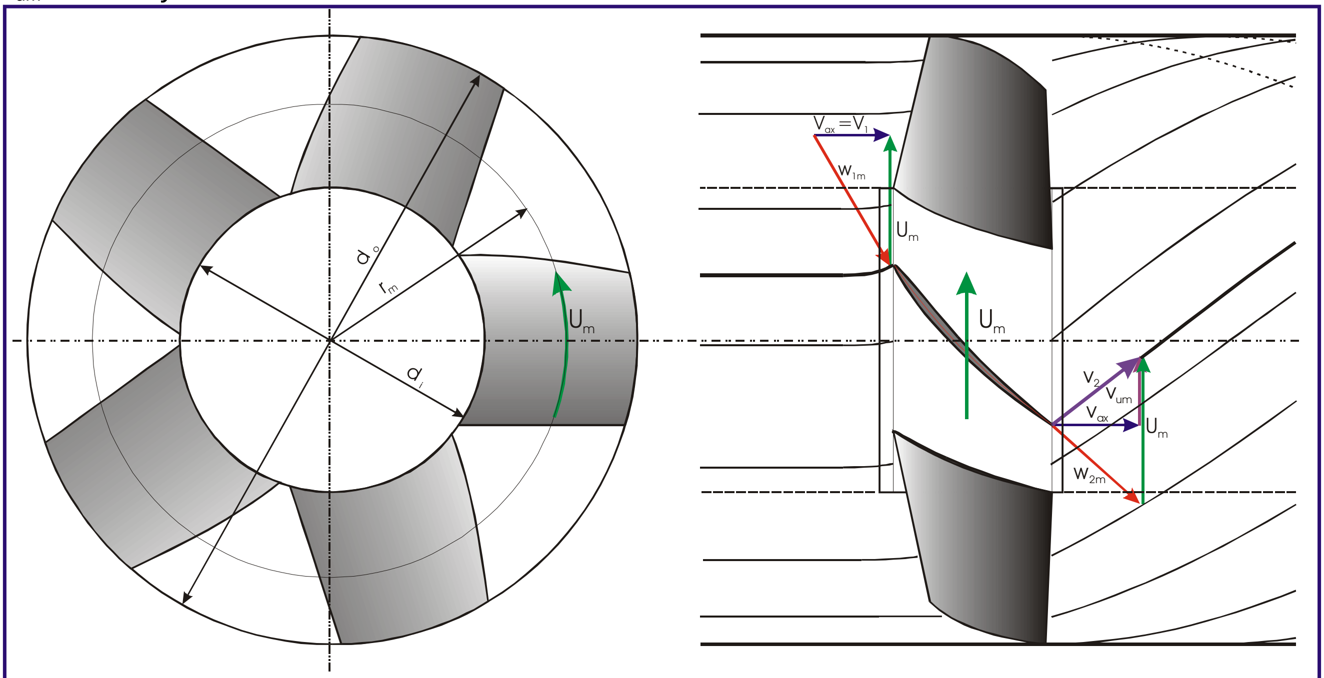
Fig.3.1

There is usually a short outer shell or casing with the motor housing fitted inside with the stator blades used as support struts. The cone of the motor housing (if supplied at all) can be separated and gives access to the motor and the wiring. The rotor is fitted to the motor shaft using adapters to account for different motor shaft diameters.
In operation air is drawn into the fan by the rotor and expelled after passing the stator blades.
By definition this arrangement is not a compressor, since its main purpose is to move air. However to accelerate the air flow to the required velocity, it has to produce at least as much pressure as is necessary to convert it into the velocity pressure of the out flowing air. This is for smaller, lower powered (500W) fans in the region of 2000Pa (N/m^2) or ca' 200mm WG (ca' 0.28 psi). At the very high end of the spectrum (above 1500W total) the pressure may well have to be as high as 5000Pa. Nevertheless there will be no sensible compression in the common sense (volume change) of the air since the pressure rise is low compared to atmospheric (100,000 Pa) and immediately converted into velocity.
In the next picture I have tried to show the airflow through a rotor. It is only meant to illustrate the principle: air flows from left to right with an axial velocity v_{ax} which remains effectively constant over the length of ducting because of the constant cross section area. Displacements on account of the finite thickness of the rotor blades are initially neglected. Because of the blade movement U_m (mean circumferential blade velocity) the leading edge of the rotor blade meets the air at the relative flow w_{1m}. The blade is angled and shaped in such a way as to allow the air to flow smoothly around it, and the curvature together with the neighbouring blade passages forces it to follow the movement of the rotor. At the rotor blade exit the air has gained a rotational component – the whirl. The new relative outflow in respect of the rotor blade is w_{2m} and the absolute exit velocity v_2. The mean whirl velocity in circumferential direction is v_{um}. This is called vorticity.

On the way from the entry to the exit of the rotating blade cascade (succession of blades), the air takes up the moment provided by the motor. The torque acting on the rotor blades is counteracted by the rate of change of momentum of the air. In other words: the moment acting on the rotor equals the product of the mass flow (through the rotor) and the change of its vorticity. Therefore the energy transfer to the air becomes:

$$P = M * U_m * v_{um} \qquad [3.1]$$

Where P = shaft power in Watt, M = mass flow in kg/s, U_m = blade speed in m/s, v_{um} = vorticity in m/s.



Development of whirl through an axial flow rotor
Some characteristic dimensions and values:

Hub diameter	: d_i	Absolute fan inlet velocity	: v_1
Fan (or casing) diameter	: d_o	Absolute fan outlet velocity	: v_2
Mean (dyn.) fan radius	: r_m	Relative fan inlet velocity	: w_{1m}
Mean circumferential velocity	: U_m	Relative fan outlet velocity	: w_{2m}
axial velocity	: v_{ax}	Mean whirl velocity	: v_{um}

Fig. 3.2

The above figure and the equation need some more explanation:

1. r_m is the dynamic mean radius of the fan annulus which is not identical with and usually 3 – 5% larger than the geometric average calculated from the inner and outer fan diameter. Its accurate value calculates to: $(1/6(d_o-d_i)*(d_i\pi+2d_o\pi)/(d_i\pi+d_o\pi))+d_i/2$.
2. U_m is the circumferential blade velocity at r_m. Obviously the blade velocity changes with the radius under consideration. It is least at the hub and largest at the tip.
3. Given that the power and the mass flow are fixed values (for a given fan design) a simple relationship exists between U (the blade speed at any radius) and the local vorticity v_u. I.e. the product U * v_u is constant. This also means that v_u changes with the radius and is least at the tip and largest at the hub.
   The same flow behaviour can be found in what is commonly called a "free vortex". A free vortex can easily be observed at the water drain from any bathtub or sink unit.

The first physicist who developed these relationships was the Swiss mathematician Leonard Euler (<u>1707 – 1783</u>) who formulated the first law of turbo machinery : "<u>The torque acting on a rotor is equal to the product of the mass flow multiplied by the change of whirl</u>".
This is equally valid for axial rotors as well as radial rotors and the reversal is also applicable to turbines.

The equation [3.1] reflects the above first law of turbo machinery, however the Euler equation in this instance is expanded by the angular velocity term to be able to deal with power rather than torque.

The next picture shows the relation in a more informative way.

However there are certain constraints which may force a designer to deviate from the "natural" free vortex flow as we will see later but for the moment we will stick with it.

An example is better than a thousand words:
From previous results we had:
d_i : 40mm, d_o: 80mm, P: 400W at 22700rpm = 378 s^{-1}

According to above and [3.1] this calculates to r_m = 31.2mm and a “whirl” = 0.808 1/s.

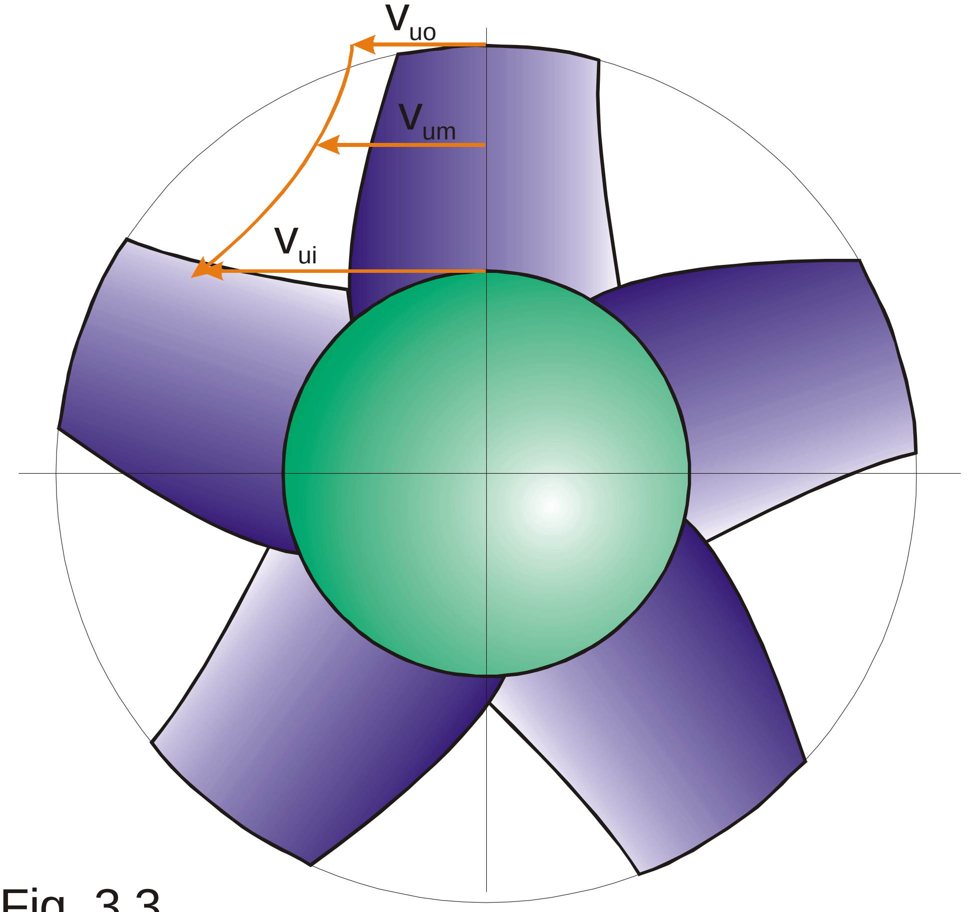


Fig. 3.3

Development of whirl velocity over fan blade radius for free vortex flow

The beauty of this equation is, that it gives us immediate access to the vorticity at any radius of the fan and allows us to calculate and draw the velocity triangles to determine the basic angles of the blade geometry at the root and tip and any intermediate station. Usually three to four stations are sufficient to define the blade angles for the whole blade. The intermediate positions are faired in.
It is quite important that the velocity triangles for any fan are determined correctly at the very beginning of the design process. They are the soul of the fan.

The next picture shows the velocity triangles again, separated from the total picture to enhance this point.

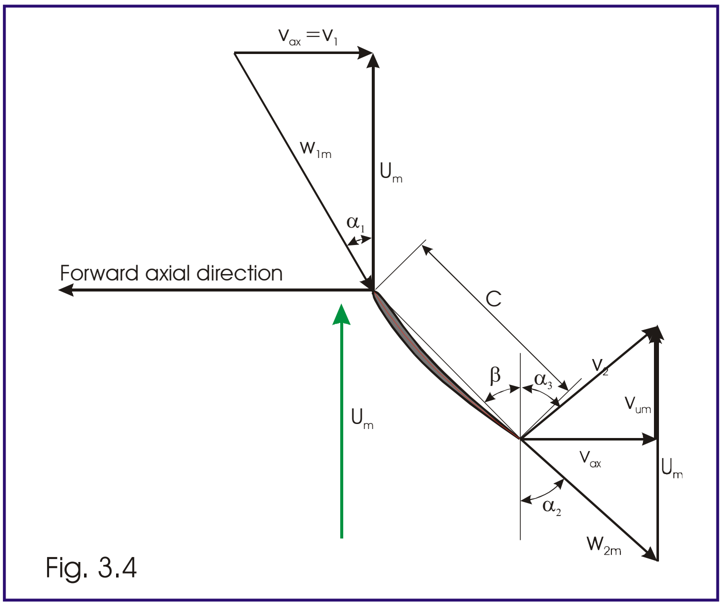


Fig. 3.4

To gain a certain understanding of the physical properties and denominations used in this context I have to explain the underlying code. I try to conform to generally accepted standards of designations but unfortunately in several cases they seem to have national idiosyncrasies and they also differ in various publications and the engineering text books.

Here (as in the previous chapters) the following letters are used for velocities, lower case “v” for absolute (*i.e.* measurable) air velocities. Lower case “w” is used as a designator for relative velocities, as in the picture above for the inlet and outlet velocities relative to the blade profile section, but it is also used in the context of the plane’s velocity relative to the air as against to the absolute speed of a (model) airplane in respect to a fixed terrestrial coordinate system. Very often I have found the letter “c” used for velocities especially in the German literature relating to multi stage compressors and turbines. It will not be used here.
The various circumferential velocities are special cases where an international consensus seems to favour a capital “U” for the absolute blade velocity and the lower case “vu” or “v_u” for the vorticity. Here “v_u” is used throughout.

Another point of content seems to be the designation of fan blade angles. It is important to define the “axis” against which an angle is measured. The blade angle can be measured against the rotational axis as is often the case in UK literature, which is then called “stagger angle” and given the designation “ξ” (Greek “zeta”). The other faction (continental Europe and –strangely- USA) prefer to measure blade angles against the peripheral direction of the rotor (90^o to the axis) for which the designation “β” (Greek “beta”) is used. That’s what is used here. So please watch out if you compare to other publications. The denomination “α” (Greek “alpha”) is used here to define the angle of the incoming and outgoing air in respect to the circumferential movement of the rotor.

There are various other notions and notations used in the literature relating to fan and compressor design. They are all extremely useful for their special purpose but not necessarily very helpful in the context of an introduction to EDF design. I will try to explain most of those and adapt the one’s which I think are relevant to our cause.

In Fig. 3.4 we can clearly see that the axial air velocity v_{ax} and the rotor velocity U_m are at right angles to each other. Drawn, as it is, as a vector diagram the resulting relative air velocity w_{1m} and the angle α_1 can either be measured with sufficient accuracy (if the scale of the vector diagram is suitable) or calculated using simple geometrical rules.

For our sample calculation U_m calculates to:

$$U_m = 2r_m / 1000\ \pi * rpm/60 = 2 * 31.2/1000 * 3.14 * 22700/60 = 74 m/s$$

The axial velocity $v_{ax} = v_1$ is the velocity which we had calculated in previous chapters as inlet velocity v_i which is for our example 45.5m/s

The relative air inlet velocity then calculates to:

$$w_{1m} = \sqrt{U_m^2 + v_1^2} = 87\text{m/s}$$

The inlet angle is derived by using the tangents formula:

$\tan \alpha_1 = v_1 / U_m$ $\alpha_1 = 31.5^o$ (for the given example!)

This settles the rotor blade inlet side.

The outlet side requires us first to calculate the circumferential whirl velocity (vorticity).
This simply is the product of the already determined whirl multiplied by the local radius:

v_{um} = whirl * r_m [3.2] = 0.808 * 31.2/1000 = 25.2m/s

We then have to draw a vector diagram as shown in Fig. 3.4 at the profile trailing edge.
Start again with v_{ax} parallel to the axial direction pointing rearwards. At the end point of that vector draw v_{um} at right angles pointing in the same direction as U_m. Now v_2 is the resulting vector. This is the absolute air velocity and direction behind the rotor. From the end point of v_{um} subtract the vector U_m. Draw another vector from the trailing edge to the origin of U_m. This resultant is the relative outlet velocity w_{2m}.

The angle α_2 calculates to:

$\tan \alpha_2 = v_{ax} / U_m - v_{um}$ $\alpha_2 = 43.2^o$ (for the given example!)

and

$$w_{2m} = \sqrt{v_{ax}^2 + (U_m - v_{um})^2} = 66.7 \text{ m/s}$$ (for the given example!)

In many text books the inlet and outlet vector diagrams are often superimposed to show the work done to the mass flow. This is a notation which I will not generally present here, but the principle may be of interest, so I show an example below:

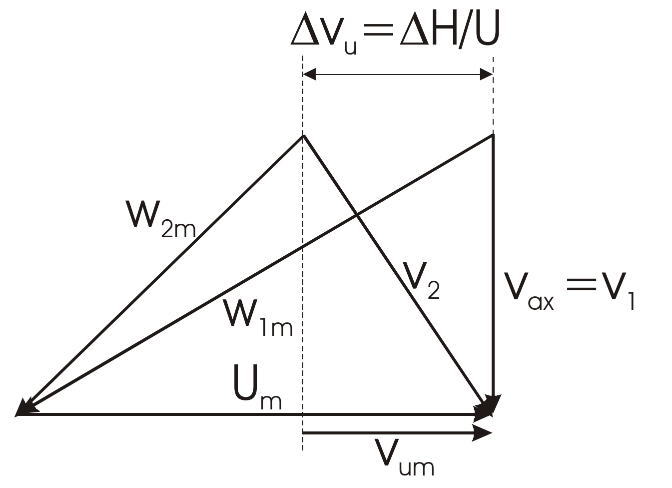


Fig. 3.5

The notation Δv_u is a more generalised term of $v_{u(m)}$ and ΔH is a general term for the energy transfer to the air flow per unit mass.
Though it is generally considered that “this form of velocity triangle is fundamental to an understanding of the design and performance of axial fans and compressors” I feel that in the

singularly specialised field of EDF design some of the denotations used (ΔH in particular) are unnecessarily complicated descriptions which can be circumvented by equally accurate but more easily understood notations. The above shown form is generally used and particularly suitable in the design of multi stage compressors where density changes and heat transfer as well as changing radii and mixed flow have to be considered.

Calculating and drawing of the velocity vectors as shown above for the radius r_m has to be performed for the other interesting sections as well, namely for the hub radius and the blade tip radius. This we will do in a later chapter where we will discuss also the Excel spread sheet program.

Pressure development in the rotor cascade

Now we will see what else happens to the air from the entrance of the rotor blades to the exit. It has been shown that the rotor introduces a whirl to the air stream which is basically axiparallel beforehand. A closer examination of the velocity vectors however reveals also that there have been changes to the velocities as well. To highlight these changes the next picture shows the developed blade cascade. This is produced (mentally) by cutting the blades off at r_m and rolling the rotor over a sheet of paper after putting some ink onto the blade ends. Instead of drawing the velocity vectors as before, here I have shown many vectors parallel to each other to imitate a form of stream lines. The left part of the illustration shows the absolute velocities v_1 and v_{2m}, the right shows the relative velocities w_{1m} and w_{2m}. Since the axial velocity is of the same value before and after the rotor it is quite obvious that v_{2m} is greater than v_1, but w_{2m} is less than w_{1m}.

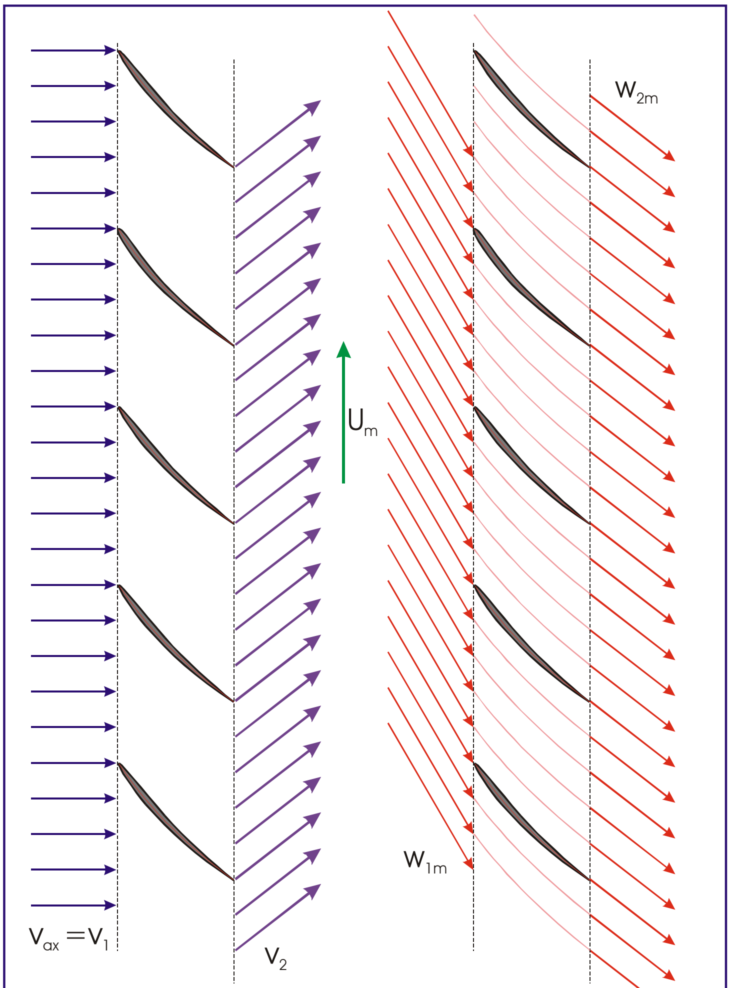


Fig. 3.6

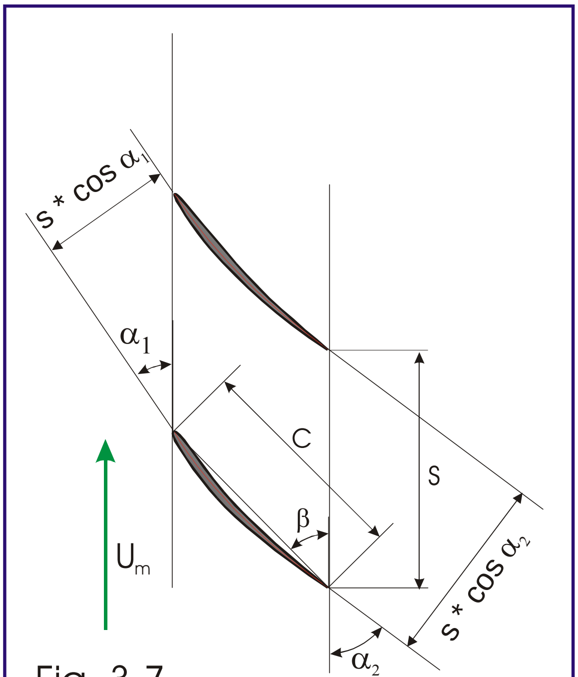


Fig. 3.7

The increase and direction of v_2 is the result of the whirl which has been created by the energy transfer in the rotor. The decrease of the relative velocity at the blade gap exit however is caused by the widening of the blade passage from the entry to the exit. The gap width at the entry calculates to

s * cos α_1 and the gap at the exit to s * cos α_2 as shown in Fig. 3.7. According to Bernoulli's law the velocity in the passage decreases and the (static) pressure increases similar to the behaviour in a diffuser.
But, as we find in a diffuser, the conversion of velocity pressure into static pressure is accompanied by losses and is also limited to certain ratios above which the efficiency suffers to an unacceptable degree. So it is in the case of the axial fan rotor, where extensive experiments and measurements by many people over the years have proved that there is a practical limit of pressure rise (per stage).
Today the ratio w_2/w_1 is generally called De Haller number and its value should not be lower than 0.7 if good efficiencies are sought.
Another notion which is often found in the literature is the pressure rise coefficient $Cp_i = 1 - (w_2/w_1)^2$ which should be kept below 0.5 to allow efficient and stall free operation of the blades.

For our sample calculation the values are:

De Haller number $w_2/w_1 = 66.7\text{m/s} / 87\text{m/s} = 0.76$
and $Cp_i = 1 - (w_2/w_1)^2 = 0.42$

The actual pressure rise in the blade passage then calculates to:

$$\Delta P = (1/2\rho w_1^2 – 1/2\rho w_2^2) - \Delta p_{Loss} \quad [3.3]$$

Where 1/2ρ is the air density / 2 and Δp_{Loss} the losses in the blade passage due to profile drag and friction of the air flow on the blades and the walls.

And for our Example:

$\Delta P = (0.6 * 9409 – 0.6 * 4449) - \Delta p_{Loss} = 5645.4 – 2669.4 - \Delta p_{Loss} = 2976\text{Pa} - \Delta p_{Loss}$

For the time being we can't put accurate values to Δp_{Loss} but they are in the region of 5% - 8%.
When IS units are used the pressure values automatically appear as N/m^2 [Pa]

Directly after the fan rotor blades the air flow has a whirl v_u as shown in Fig. 3.4 for the mean radius. The flow therefore has an angle α_3 in the direction of the rotor rotation which means that there is still some velocity energy left in the air flow which may be converted to (static) pressure if the whirl could be eliminated. This is given to a row of guide vanes after the rotor which is called the stator.
Using simple geometry the angle is given by:

$tg\alpha_3 = v_{um} / v_{ax} = 25.2 / 45.5 = 1.8 \quad \alpha_3 = 29^o$ (for the given example!)

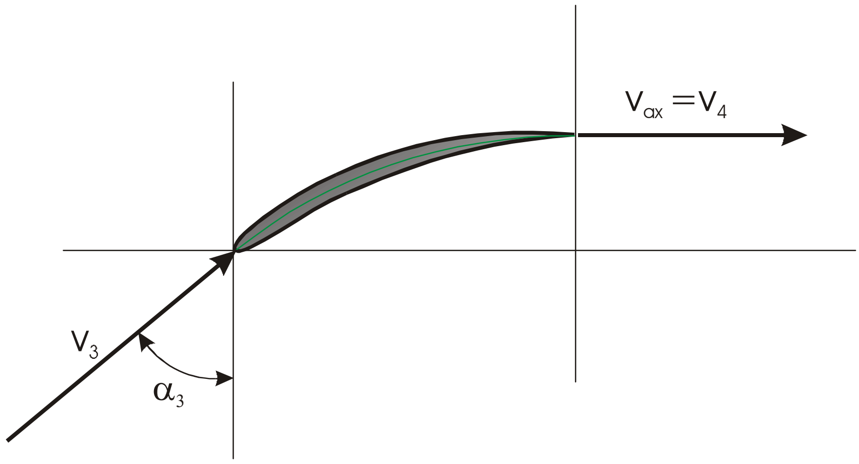


Fig. 3.8

Fig. 3.8 shows the arrangement of a single stator blade at the r_m section. It is obvious in this picture that the air flow after the stator is again without any whirl and the velocity of v_4 is the same as v_{ax}, at least that is what we are aiming for. One can also see clearly that v_4 is less than v_3.
Assuming for the moment that v_4 is of the same value as v_{ax} the velocity of v_3 can be derived geometrically:

$$v_3 = v_4 / \cos 90^o\text{-}\alpha_3 \quad = 45.5 / 0.875 = 52\text{m/s} \quad \text{(for the given example!)}$$

So in the case of this stator and neglecting the losses

$$\Delta P_{stator} = (1/2 \rho v_3^2 - 1/2 \rho v_4^2)$$

$$\Delta P_{stator} = (0.6 * 2706 - 0.6 * 2070) = 1623.6 - 1242 = 381\text{Pa}$$

The total pressure development of rotor and matching stator for our sample EDF (at the mean radius r_m) then will be 2976Pa + 381Pa = 3357Pa, discounting for the inevitable losses!
It is generally acknowledged that the incorporation of matched stators is advisable when the whirl angle 90^o - α_3 is more than 15^o. If the whirl angle is less than that the complications and expense of cambered stator profiles are often not justified. This is particularly true for those small and low powered Far East toy products which work surprisingly well (but are not really the subject of this article).
There are two more aspects to the issue of outlet guide vanes (OGWs). As one can deduct from the above, a design which leaves a “rest” whirl after the stator which is not greater than around 5 - 10^o is quite acceptable from a performance point of view. And the maximum deflection angle which can be safely achieved in a stator is 45^o. This will become more acute later in this treatise when not only mean radius positions will be considered but also the air flow behaviour near and at the hub.