$F_c =$ Centripetal force (relative to airspeed vector)

$\theta =$ heading $\angle$ between airspeed and $y$ axis

- Heading increases at a constant rate as airspeed vector rotates clockwise at constant rate.
- Rate of rotation of groundspeed vector is not constant.
- We're focusing especially on the case where wind is causing a decrease in both aspd $r$ ground, and heading has not yet reached straight-into-wind. I.e., the upper right quadrant of the coordinate grid.

- Wind is constant and blowing purely east-to-west.
- The illustrated scale relationship between $F_c$ and the various speed vectors is arbitrary.
- Wind vector merely translates, does not rotate.
Assume - Airspeed + Fc vectors constant in magnitude but rotating clockwise at constant rate =
angular velocity = \frac{\text{d} \text{h} \text{e} \text{n} \text{d} \text{i} \text{n} \text{g}}{\text{d} \text{t} \text{i} \text{m} \text{e}}

- the groundspeed vector is the vector sum of
airspeed vector + windspeed vector

- windspeed vector is constant in magnitude +
direction.

Working from the aspd-gs-wv vector triangle,
We'll break the aspd + gs vectors down into x + y components, and take derivatives of
these components to find acceleration components
based on assumption of fixed aspd + fixed
_heading. We'll compare these to the acceleration
of
components yielded by breaking Fc down into
x+y components,

- note that no matter how heading changes,
wind acts only in the X-axis.

- We'll explore whether the x + y components of Fc
are adequate to explain all the acceleration taking
place on the aspd vector, and the groundspeed
vector, as the heading changes at a constant
rate, and the aspd vector stays constant in
magnitude, and the aspd-wv-gs vector triangle stays intact.
\[ x \text{ coordinate of } \text{ground speed} = (x \text{ coord of as pd}) - \text{ wind} \]

\[ y \text{ coordinate of } \text{ground speed} = y \text{ coord of as pd} \]

\[ x \text{ coord as pd} = \text{as pd} \cdot \sin \text{ heading} \]

\[ y \text{ coord as pd} = \text{as pd} \cdot \cos \text{ heading} \]

\[ x \text{ coord ground speed} = (\text{as pd} \cdot \sin \text{ heading}) - \text{ wind} \]

\[ y \text{ coord ground speed} = \text{as pd} \cdot \cos \text{ heading} \]

- Now we take the derivatives -

\[ \frac{\Delta}{\Delta t} (x \text{ coord ground speed}) = \frac{\Delta}{\Delta t} ((\text{as pd} \cdot \sin \text{ heading}) - \text{ wind}) \]

\[ = \frac{\Delta}{\Delta t} (\text{as pd} \cdot \sin \text{ heading}) \]

\[ \text{(Use "h" for heading, "w" for wind)} \]

\[ = (\frac{\Delta}{\Delta t} (\text{as pd})), \sin h + (\frac{\Delta}{\Delta t} (\sin (h))) \text{ as pd} \]

\[ = 0 + \frac{\Delta}{\Delta t} (\sin (h)) \text{ as pd} \]

\[ \text{because as pd is constant} \]

\[ = \text{as pd} \cdot \frac{\Delta}{\Delta h} (\sin (h)) \cdot \frac{\Delta h}{\Delta t} \]

\[ \text{\textbf{\textit{\textbf{\Delta Note: This is also}}}} \]

\[ \frac{\Delta}{\Delta t} (x \text{ coord as pd}) \]

\[ \frac{\Delta}{\Delta t} (y \text{ coord as pd}) = \frac{\Delta}{\Delta t} (\text{as pd} \cdot \cos (h)) \]

\[ = \text{as pd} \cdot \frac{\Delta}{\Delta t} (\cos (h)) \]

\[ = \text{as pd} \cdot \frac{\Delta}{\Delta h} (\cos (h)) \cdot \frac{\Delta h}{\Delta t} \]

\[ = \text{as pd} \cdot (-\sin (h)) \cdot \text{angular velocity} \]

\[ \text{\textbf{\textit{\textbf{\Delta Note: This is also}}}} \]

\[ \frac{\Delta}{\Delta t} (y \text{ coord as pd}) \]
Summarizing - From the wind-aspd-gspd vector triangle, we find:

\[
\frac{\partial}{\partial t} (x \text{ coord gspd}) = \frac{\partial}{\partial t} (x \text{ coord aspd}) = \text{ aspd} \cdot \cos (h) \cdot \text{ angular velocity}
\]

\[
\frac{\partial}{\partial t} (y \text{ coord gspd}) = \frac{\partial}{\partial t} (y \text{ coord aspd}) = \text{ aspd} \cdot (-\sin (h)) \cdot \text{ angular velocity}
\]

Now - what are the \( x \) \& \( y \) coordinates of \( F_C \)?

The heading vector is parallel to the airspeed vector. 
\( F_C \) points \( 90^\circ \) ahead (clockwise) of airspeed \& heading.
\( 90^\circ = \frac{\pi}{2} \) radians.

The angle between \( y \) axis (north) \& \( F_C \) is \( (\text{heading} + \frac{\pi}{2}) \)

\[
\begin{align*}
&\text{X coord of } F_C \text{ is } \sin \left( \frac{\pi}{2} + \text{heading} \right) \cdot F_C, \text{ resulting acceleration is} \\
&\text{Y coord of } F_C \text{ is } \cos \left( \frac{\pi}{2} + \text{heading} \right) \cdot F_C, \text{ resulting acceleration is } \cos \left( \frac{\pi}{2} + \text{heading} \right) \cdot \frac{F_C}{m} \\
\end{align*}
\]

\[\text{Trig identities: } \sin (\frac{\pi}{2} + x) = \cos x, \ \cos (\frac{\pi}{2} + x) = -\sin x\]

So, \( X \) coord of \( F_C \) is \( \cos (h) \cdot F_C \), resulting acceleration is \( \cos (h) \cdot \frac{F_C}{m} \).

\( Y \) coord of \( F_C \) is \( -\sin (h) \cdot F_C \), resulting acceleration is \( -\sin (h) \cdot \frac{F_C}{m} \).

* - If we prefer, there is a way to get these expressions using similar triangles rather than the unit circle.
Summarizing -

Working from the vector triangle of aspd, gspd, t wind, we find:

A) \( \frac{\dot{d}}{dt} \times \text{coord gspd} = \frac{\dot{d}}{dt} \times \text{coord aspd} \times \text{aspd} \times \cos(h) \times \text{angular velocity} \)

B) \( \frac{\dot{d}}{dt} \times \text{coord gspd} = \frac{\dot{d}}{dt} \times \text{coord aspd} \times \text{aspd} \times (-\sin(h)) \times \text{angular velocity} \)

Working directly from \( F_c \) we find -

\( F_c \) causes an acceleration with -

C) \( x \text{ component of } \cos(h) \times \frac{F_c}{m} \)

D) \( y \text{ component of } (-\sin(h)) \times \frac{F_c}{m} \)

If \((\text{airspeed times angular velocity}) = \frac{F_c}{m}\), then

expressions A & C are identical,
expressions B & D are identical.

This illustrates that all the acceleration acting on the airspeed vector, and all the acceleration acting on the groundspeed vector, as the heading vector rotates around at a constant rate, is the vector triangle of aspd, gspd, t wind is maintained, is solely attributable to the action of \( F_c \).

Is \((\text{airspeed times angular velocity}) = \frac{F_c}{m}\)?
Is \( \text{angular velocity} = \frac{F_c}{m} \) ?

\[ F_c = \frac{mv^2}{v} \]

Time per circle = \( \frac{2\pi r}{v} = 2\pi \frac{mv^2}{F_c} = 2\pi \frac{m}{F_c} v \)

Circles per time = \( \frac{F_c}{2\pi mv} \)

\[ \frac{\text{radians}}{\text{time}} = \frac{\text{circles}}{\text{time}} \cdot \frac{2\pi \text{ radians}}{\text{circle}} = \frac{F_c}{mv} \]

YES

- \( F_c \) is sufficient to produce all the observed change in the axial speed vector and the groundspeed vector.

We could have reached the same conclusion simply by noting that in the airmass reference frame, if the magnitude of the axial speed vector is defined to be constant, then \( F_c \) must be the only force present (at least in the horizontal plane).

If \( F_c \) is the only force present in one given inertial reference frame, then it must be the only force present in any other valid inertial reference frame, e.g. the groundspeed reference frame.

- Bear in mind the wind is a velocity, not a force!
― Bear in mind the wind is a velocity, not a force! —

Recall that a net acceleration vector, and likewise a net force vector, are the same magnitude and direction in any valid inertial reference frame, i.e., any non-accelerating reference frame.

This is not true of velocity vectors.

If we know the net force and net acceleration vector in one inertial reference frame,

then we know them in all inertial reference frames.

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- If the groundspeed vector were experiencing a different acceleration than the airspeed vector, this would mean that the aircraft would have a different acceleration vector as seen in the airmass reference frame than as seen in the ground reference frame. This is only possible if the airmass reference frame is accelerating relative to the ground reference frame, and/or vice versa. But this cannot be the case if both the ground reference frame and the airmass reference frame are both valid inertial (unaccelerated) reference frames, and neither is privileged over the other.

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What if it were somehow true that our initial vector diagram accurately showed the orientation of the net force vector with respect to the airspeed and groundspeed vectors, but the airspeed and groundspeed vectors somehow began to experience a different net acceleration than shown here \( \left( \frac{F_c}{m} \right) \) as the heading rotated further into the wind? That could only happen if some additional acceleration vector were coming into play. As long as the wind vector is constant, there is no such additional acceleration vector. Note that "inertial" constitutes neither a force nor an acceleration.

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Just imagine what an aid it would be to
designers of inertial navigation systems, if
we could deduce the direction and speed
of an unvarying wind just by noting
changes in net acceleration, and in the
magnitude of the airspeed vector, while
flying constant-banked circles. This
is not the case in reality.

There's magic in the flight of 14
at Barross-

magic generated by the wind gradient

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Addendum—note that our work in the middle or bottom of page 4, breaking down \( F_c \) into \( x \times y \) components made no assumption that \( F_c \), airspeed, rate of heading change, or any other variable was constant.

Nor did we explicitly assume that no other force or acceleration component existed other than \( F_c \) - these results are valid for instantaneous values of \( F_c \times y \) components even if airspeed is increasing or decreasing due to excess thrust or drag, etc.

The same is true of our work on p. 6 where we show that airspeed angular velocity = \( \frac{F_c}{m} \).

In contrast our work on page 2, taking derivatives of the \( x \times y \) components of groundspeed \& airspeed vectors as derived from the wind-groundspeed-airspeed vector triangle, does assume constant airspeed.

This is what led to the result that the \( x \) and \( y \) components of \( F_c \), as derived on p. 4, were the only acceleration components at play.

The work on page 3 essentially shows that if 1 of these things are true, then ALL are true:

1. Airspeed is constant in magnitude
2. \( F_c \) is the only force (\textit{ie} the net force at play)
3. Rate of heading change is constant
Addendum Ltd -

The work on page 5 essentially shows that if
1 of these things are true, then ALL are true:

(1) Airspeed is constant in magnitude

(2) Fc is the net force. No additional forces
    should be added to the force vector diagram
    as seen in the horizontal plane.

(3) Rate of heading change is constant.

If we wish to argue that airspeed is not constant
in magnitude, then we MUST show what
force, other than Fc, is causing that
variation. We cannot set net force = Fc.

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